Pathria 2.4 $H = \frac{M^2}{2I}$

$M^2 = (M_\theta^2 + M_\phi^2) \Rightarrow$ Volume in phase $= M^2 (2\pi)^2$,

The $(2\pi)^2$ factor comes from $\theta \in [0, 2\pi]$, $\phi \in [0, 2\pi]$.

Each state occupies $h \Rightarrow$ # microstates below M is given by $\frac{(2\pi)^2 M^2}{h^2} = \left(\frac{M}{\hbar}\right)^2$.

For $M = \sqrt{j(j+1)}\hbar$, $\left(\frac{M}{\hbar}\right)^2 = j(j+1)$

the degeneracy is then given by $\left(\frac{M_j}{\hbar}\right)^2 - \left(\frac{M_{j-1}}{\hbar}\right)^2 = 2j$

If $j = 0, 1, 2, \ldots$, the degeneracy is $0, 2, 4, \ldots$

If $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$, the degeneracy is given by $1, 3, 5, 7, \ldots$

Clearly, the latter is the more agreeable degeneracy since in quantum mechanics, we know the degeneracy is given by projection onto the z-axis, which gives $1, 3, 5, 7, \ldots$.